Dynamics of Hollow Elliptical Cylinder Arrays

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Strain-softening materials have gained significant interest in the physics, mathematics and material science communities recently. Most materials we are familiar with are strain-hardening, meaning as they deform, they become harder to deform. Strain-softening materials have the opposite property; as they deform, they become easier to deform. The dynamics of strain-softening materials in an experimental setting have been rarely studied. I have been working on modeling and simulating a system that exhibits such strain-softening behavior. It consists of a chain of hollow elliptical cylinders (HEC). An initial force on the first particle of the chain puts a dispersive shock wave in motion. A dispersive shock wave through strain-softening material has a peak that propagates slower than the rest of the wave, causing it to eventually disperse. The point at which the wave breaks causes sudden changes in its surrounding atmosphere and can be very damaging. Thus, mathematically modeling this break will help prepare for such an effect. Using an ordinary differential equation found on the basis of spherical particle interactions through the Hertzian contact law describes the motion of the wave. My initial research was understanding and deriving this equation.

Later, I tailored the mathematical model to experimental results conducted at the University of Washington, where I researched for three with my advisor's collaborator, Professor Jinkyu Yang. We considered a few different HEC models – a monomer, the particles have the same masses, a dimer, where the masses alternate, and a trimer, where three different masses alternate. For the monomer model, I found coefficients for the chain's stiffness, linearity, and damping that better fit the amplitude and speed of the experiment (Figure 1). I also tried finding a best-fit curve for a symmetric and generalized dimer model that relates the frequency of wave's oscillations to the wave number using optimization techniques. This dispersion curve helps explain how accurate our model is. Additionally, in order to verify that we are observing a dispersive shock wave, and not some other wave, the wave speed must depend on its amplitude. So as the wave slows, the amplitude decreases, demonstrating that the wave disperses. I modeled this particular graph (Figure 2).

To conclude the results of my research, we now have a very good monomer model that replicates a dispersive shock wave and have made progress for the dimer model.

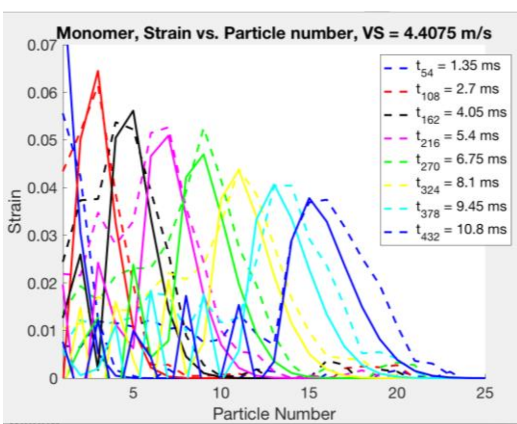

Figure 1

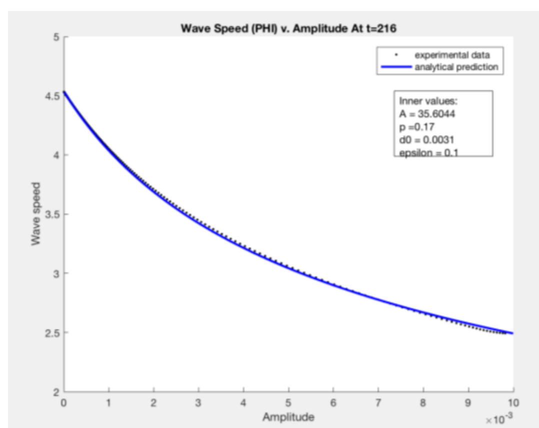

Figure 2

Faculty Mentor: Professor Christopher Chong
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